RANS Modeling of Stably Stratified Turbulent Boundary Layer Flows in OpenFOAM

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Talk Outline

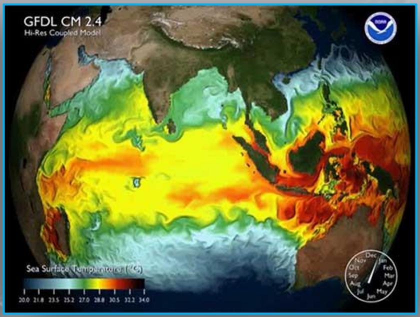
- Motivation
- Theoretical Background
- Numerical Setup
- Simulation Results
- Conclusions
- Future Research

Motivation

- The atmospheric boundary layer (ABL) is a very turbulent flow $Re \sim O(10^7)$
- Turbulent processes influence the transport of momentum, heat, humidity, and scalars.

Wind Energy

Weather

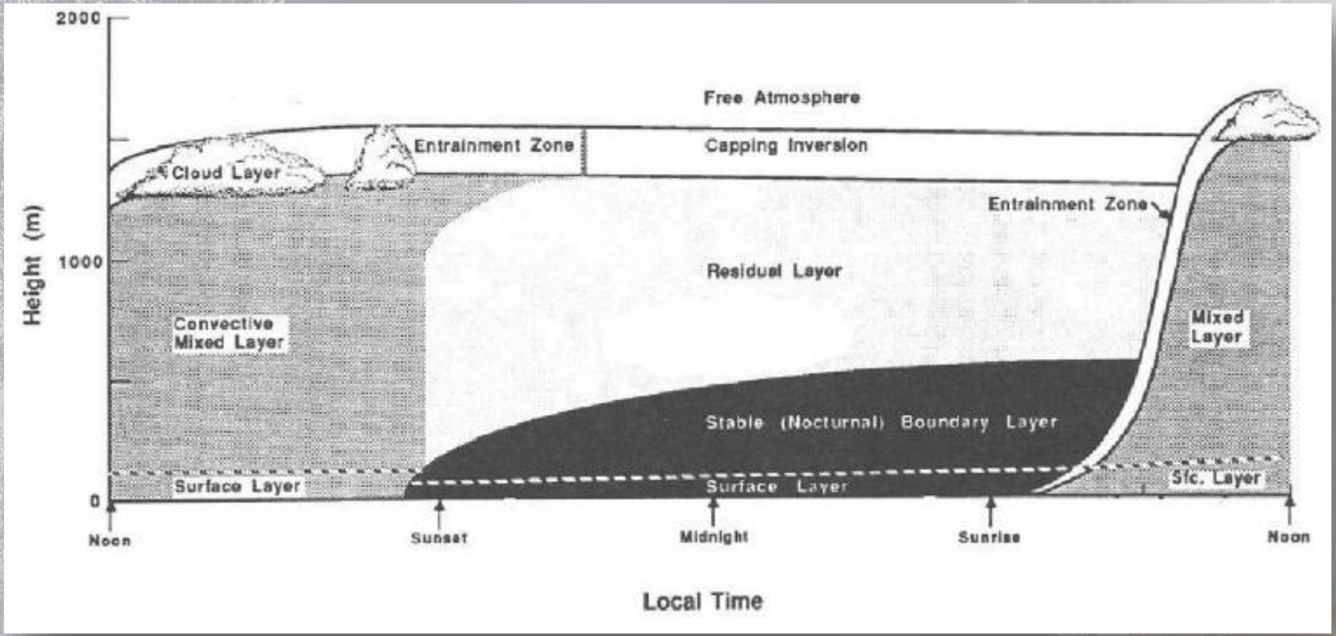


Climate Change

Air Quality

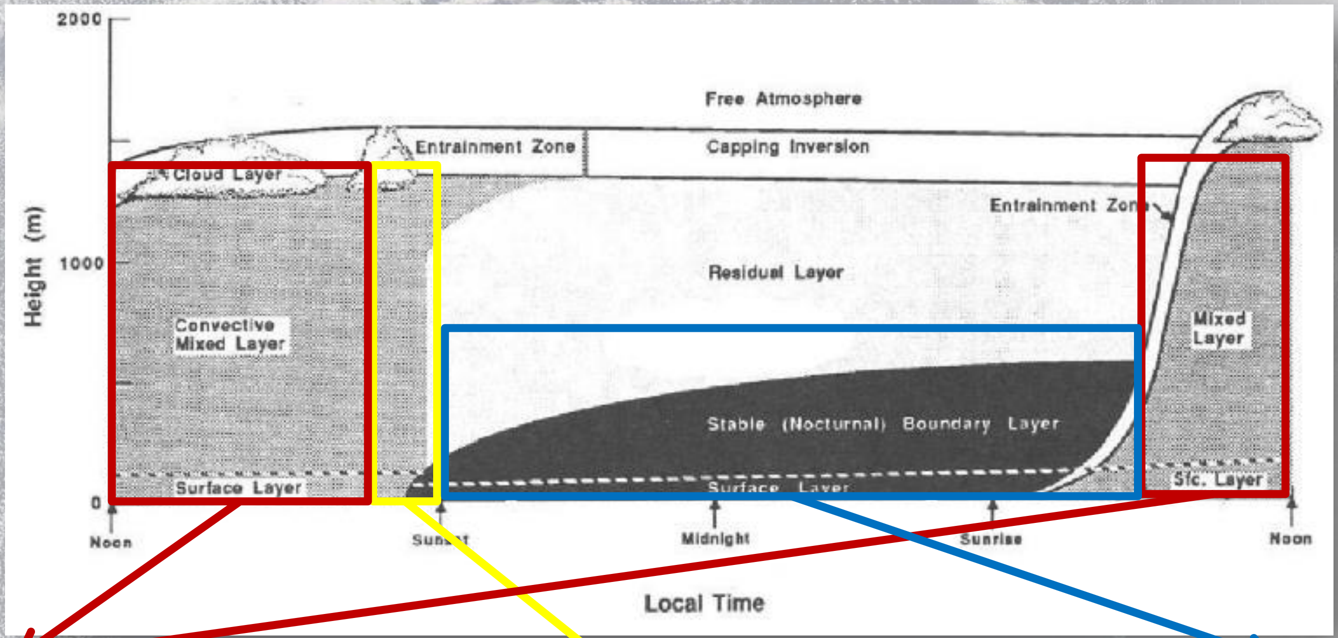
Motivation

- The ABL experiences a diurnal cycle (~24 hr) due to the radiative heating and cooling of the earth's surface



Stull (1988)

Motivation



Stull (1988)

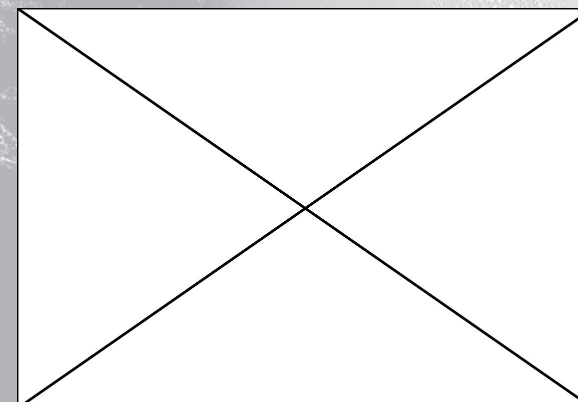
Unstable ABL
- Positive surface heat flux
- Mixing is enhanced by convection
- Surface, free convection, and mixed layers

Neutral ABL
- Uniform temperature distribution
- Logarithmic wind profile
- Relatively well understood

Stable ABL
- Negative surface heat flux
- Mixing is suppressed resulting in a shallow layer
- Exhibit inertial phenomena (e.g. jets and waves)

Motivation

- The Stable Atmospheric Boundary Layer (SABL)
  - Buoyancy forces lead to:
    - Wave formation and breaking (intermittent turbulence)
    - Low-level jets
    - Suppressed turbulent scales
  - These effects present significant challenges for wind energy
    - Increased shear forces and fatigue loading on wind turbines



Theoretical Background

- **Reynolds-Averaged Navier-Stokes (RANS) Framework**
  - **Continuity**
    - $\frac{\partial \overline{U}_j}{\partial x_i} = 0$
  - **Momentum**
    - $\frac{\overline{D}\overline{U}_j}{\overline{D}t} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_i}\left[\nu_{eff}\left(\frac{\partial \overline{U}_i}{\partial x_j} + \frac{\partial \overline{U}_j}{\partial x_i}\right)\right] + g\delta_{ij3} + 2\Omega_{ij} \times \overline{U}_{ij} + F_j$
    - $\nu_{eff} = \nu + \nu_t$
  - **Scalar (Density) Transport**
    - $\frac{\overline{D}\overline{\rho}}{\overline{D}t} = \frac{\partial}{\partial x_i}\left(\kappa_{eff}\frac{\partial \overline{\rho}}{\partial x_i}\right)$
    - $\kappa_{eff} = \kappa + \kappa_t \quad \Rightarrow \quad \kappa_t = \frac{\nu_t}{Pr_t}$

Theoretical Background

- **Quantifying Stability**
  - **Gradient Richardson Number (Ri)**
    - $\mathrm{Ri} = \frac{N^2}{S^2}$
    - Where $N = \sqrt{-\left(\frac{g}{\rho_0}\right)\frac{d\bar{\rho}}{dz}}$ is the buoyancy frequency and $S = \frac{d\bar{u}}{dz}$ is the mean shear
  - **Flux Richardson Number ($\mathrm{Ri_f}$)**
    - $\mathrm{Ri_f} = \frac{-B}{P}$
    - Where $B = -\left(\frac{g}{\rho_0}\right)\overline{w'\rho'} = \kappa_t\left(\frac{g}{\rho_0}\right)\frac{d\bar{\rho}}{dz}$ is the buoyancy production and $P = -\overline{u'w'}S = \nu_t S^2$ is the shear production
  - **Monin-Obukhov length**
    - $L = -\frac{u_*^3}{\left(\frac{g}{\rho_0}\right)\kappa\left(\overline{w'\rho'}\right)_s}$

Theoretical Background

- **$\mathbf{k - \varepsilon}$ Turbulence Model**
  - Shown to perform well for stably stratified geophysical flows (e.g. Rodi 1987; Baumert & Peters 2000) and SABL (Detering & Etling 1985; Apsley & Castro 1997)

- **Turbulent kinetic energy (k)**
  - $\frac{\bar{D}k}{\bar{D}t} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right) + P + B - \varepsilon$
- **Dissipation rate of turbulent kinetic energy (ε)**
  - $\frac{\bar{D}\varepsilon}{\bar{D}t} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right) + C_{\varepsilon 1}(P + C_{\varepsilon 3}B)\frac{\varepsilon}{k} - C_{\varepsilon 2}\frac{\varepsilon^2}{k}$
- **Turbulent viscosity (ν_t)**
  - $\nu_t = (1 - Ri_f)C_\mu \frac{k^2}{\varepsilon}$

σ_k	σ_ε	C_μ	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	$C_{\varepsilon 3}$
1.0	1.3	0.09	1.44	1.92	-1.44

Theoretical Background

- **The Turbulent Prandtl Number (Pr_t)**
  - Links eddy viscosity and diffusivity in a RANS framework
  - In modeling, generally assumed to be a constant ($\mathrm{Pr_t} = 0.85$; e.g. Wilcox 1994)
  - However, research has shown that Pr_t is strongly linked to stability for stable stratification

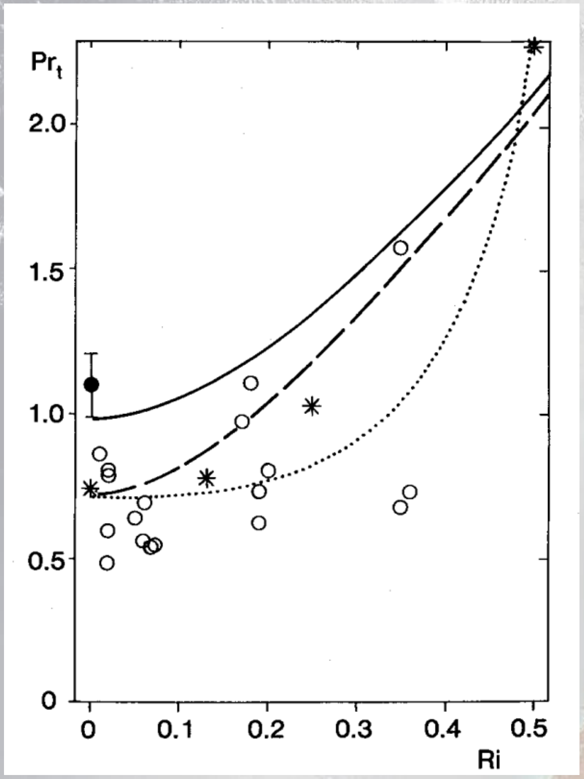
  - Firstly, we will consider 3 Pr_t formulations
    - *A constant value*: $\boxed{\mathrm{Pr_t} = 0.85}$
    - *Kim & Mahrt (1992)* develop a $\mathrm{Pr_t}$ from the Louis (1981) model given by:

$$Pr_t = \frac{l_{0,m}^2}{l_{0,h}^2}\frac{\phi_m^2(Ri)}{\phi_h^2(Ri)}$$

$$\Rightarrow \quad \boxed{\mathrm{Pr_t} = \mathrm{Pr_{t0}}\frac{1+15\mathrm{Ri}(1+5\mathrm{Ri})^{1/2}}{1+10\mathrm{Ri}(1+5\mathrm{Ri})^{-1/2}} \qquad \textbf{(KM92)}}$$

Theoretical Background

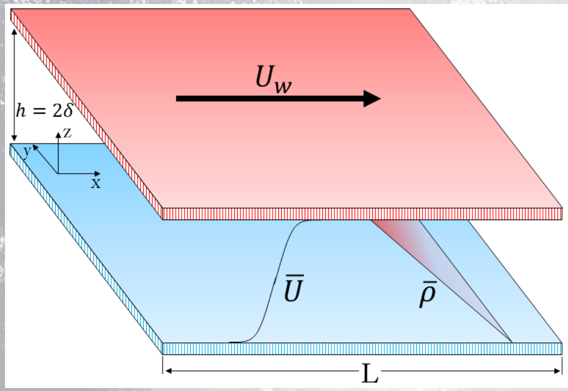
- **The Turbulent Prandtl Number (Pr_t)**
  - Venayagamoorthy & Stretch (2010) developed a $\mathrm{Pr_t}$ formulation for stably stratified homogeneous shear flows from the empirical model of Schumann & Gerz (1995)
  - Sought to generalize a Prandtl number formulation to include irreversible contributions and the neutral value of $\mathrm{Pr_t}$
  - For the weakly stratified regime ($\mathrm{Ri} \lesssim 0.25$)
    - $\mathrm{Pr_t} = \mathrm{Pr_{t0}} + \mathrm{Ri}$
  - For the strongly stratified regime (large Ri)
    - $\mathrm{Pr_t} = \frac{1}{\mathrm{Ri_{f\infty}}}\mathrm{Ri}$
  - Fitting a blending function between these two regimes results in:
    - $\boxed{\mathrm{Pr_t} = \mathrm{Pr_{t0}} \exp\left(-\frac{\mathrm{Ri}}{\mathrm{Pr_{t0}}\Gamma_\infty}\right) + \frac{\mathrm{Ri}}{\mathrm{Ri_{f\infty}}} \quad \textbf{(VS10)}}$
      - $\mathrm{Pr_{t0}} \simeq 0.7$
      - $\Gamma = \frac{Ri_f}{(1-Ri_f)} \quad \Rightarrow \quad Ri_{f\infty} \simeq 0.25 \quad \Rightarrow \quad \Gamma_\infty = 1/3$



Schumann & Gerz (1995)

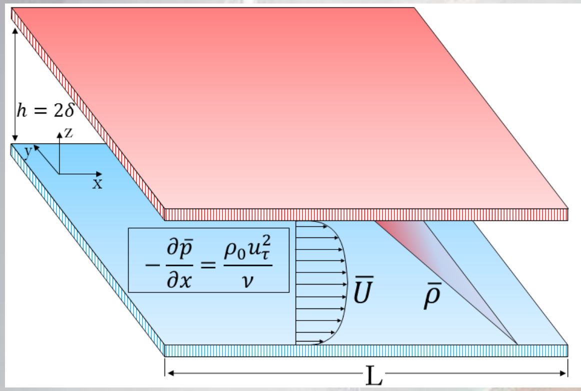
Numerical Setup

- **Starting with 2 simplified flow cases**
  - **Stably stratified turbulent Couette flow**
    - Based on the DNS work of Garcia-Villalba et al. (2011a)
      - $Re_\tau = 540, Ri_\tau = 83.5$
      - $Re_\tau = 540, Ri_\tau = 167$



$$Re_\tau = \frac{u_\tau \delta}{\nu}$$

  - **Stably stratified turbulent channel flow**
    - Based on the DNS work of Garcia-Villalba & del Alamo (2011b)
      - $Re_\tau = 550, Ri_\tau = 60$
      - $Re_\tau = 550, Ri_\tau = 120$



$$Ri_\tau = \frac{\Delta \rho g h}{\rho_0 u_\tau^2}$$

Numerical Setup

- OpenFOAM
  - Couette flow

```
tmp<fvVectorMatrix> UEqn
    (
        fvm::div(phi, U)
      + turbulence->divDevReff(U)
      ==
        sources(U)
    );

    Ueqn.relax();

    sources.constrain(UEqn());

    UEqn().solve();
```

Numerical Setup

- OpenFOAM
  - Channel flow

```
tmp<fvVectorMatrix> UEqn
    (
        fvm::div(phi, U)
      + turbulence->divDevReff(U)
      ==
        sources(U)
    );

    UEqn().relax();

    sources.constrain(UEqn());

    solve(UEqn() == -gradP);
```

Numerical Setup

- OpenFOAM
  - Density transport equation

```
kappat = turbulence->nut()/Prt;
kappat.correctBoundaryConditions();
volScalarField kappaEff(“kappaEff”, turbulence->nu()/Pr + kappat);

fvScalarMatrix rhoEqn
(
    fvm::div(phi, rho)
   -fvm::laplacian(kappaEff,rho)
);

rhoEqn.relax();
rhoEqn.solve();
```

Numerical Setup

OpenFOAM
Modified $k - \varepsilon$ turbulence model

Lookup mean density field

```
rho.db().lookupObject<volScalarField>("rho");
```

Calculate density gradient

```
gradRho_ = fvc::grad(rho_);
gradrho_ = gradRho_.component(2);
```

Calculate buoyancy frequency (squared)

```
N2_ = (-g_/rho0_)*gradrho_;
```

Calculate gradient Richardson number

```
Rig_ = (N2_/(2.0*magSqr(symm(fvc::grad(U_)))));
```

Calculate buoyancy production

```
B_ = (nut_/Prt_)*(g_/rho0_)*grahrho_;
```

Calculate flux Richardson number

```
Rif_ = min(1.0,-B_/max(Gk_,epsilonMin_));
```

Modified ε equation

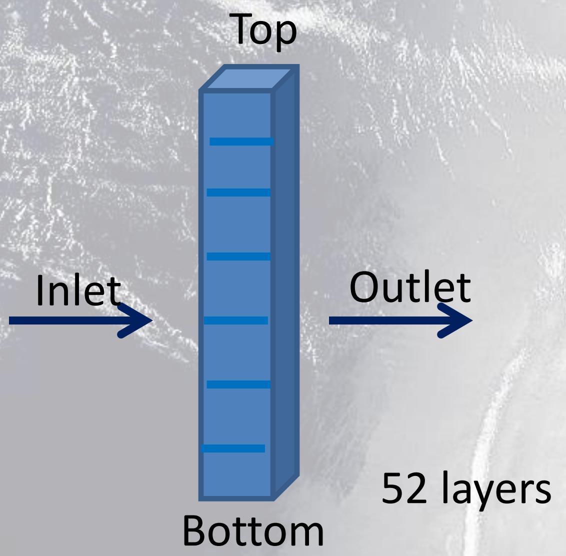
```
… C1_*(G + C3_*B))*epsilon_/k_ …
```

Modified ν_t equation

```
nut_ = (1.0 – Rif_)*Cmu_*sqr(k_)/epsilon_;
```

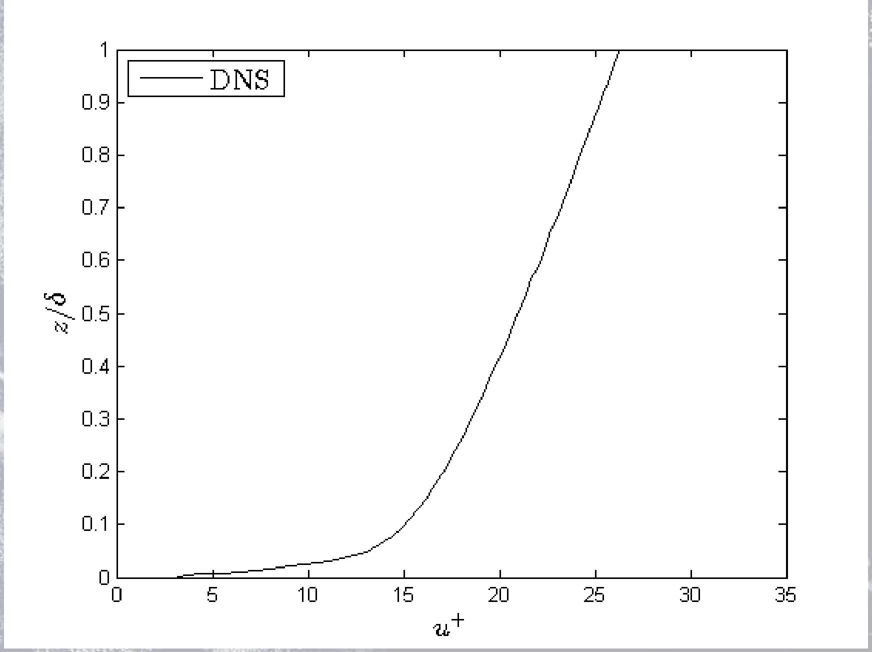
Numerical Setup

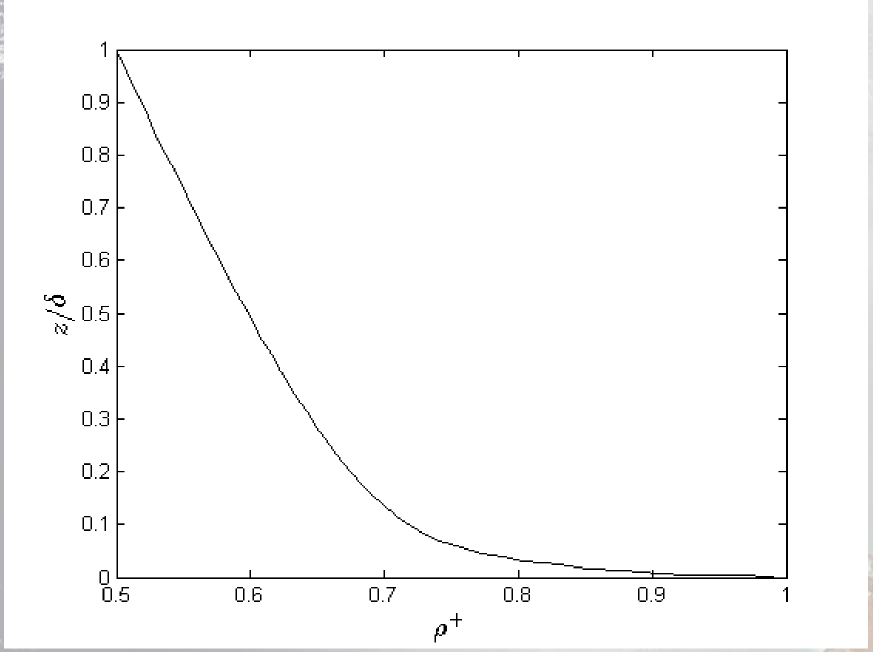
- OpenFOAM
  - Boundary conditions
    - Inlet and outlet – cyclic
    - Front and back – empty
    - Top and bottom – walls
  - Standard wall functions
    - nutkWallFunction
    - kqRWallFunction
    - epsilonWallFunction
  - Second-order accuracy discretization



Couette Flow Results

- $Ri_\tau = 83.5$





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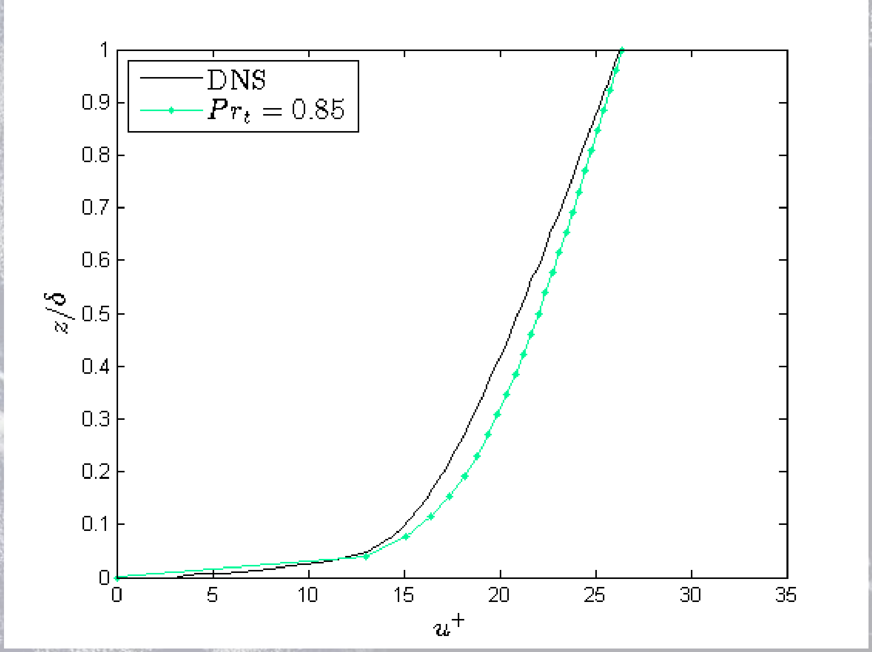

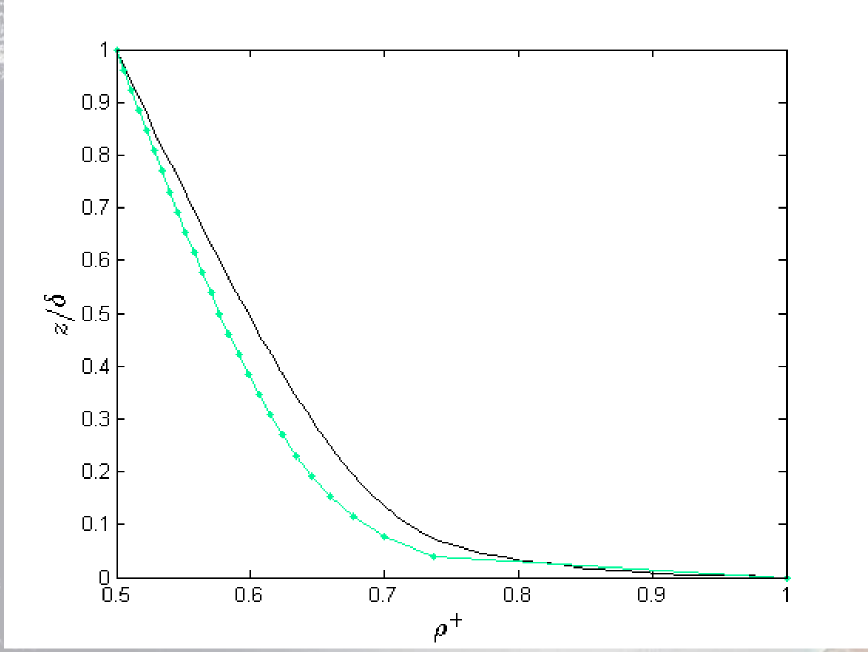

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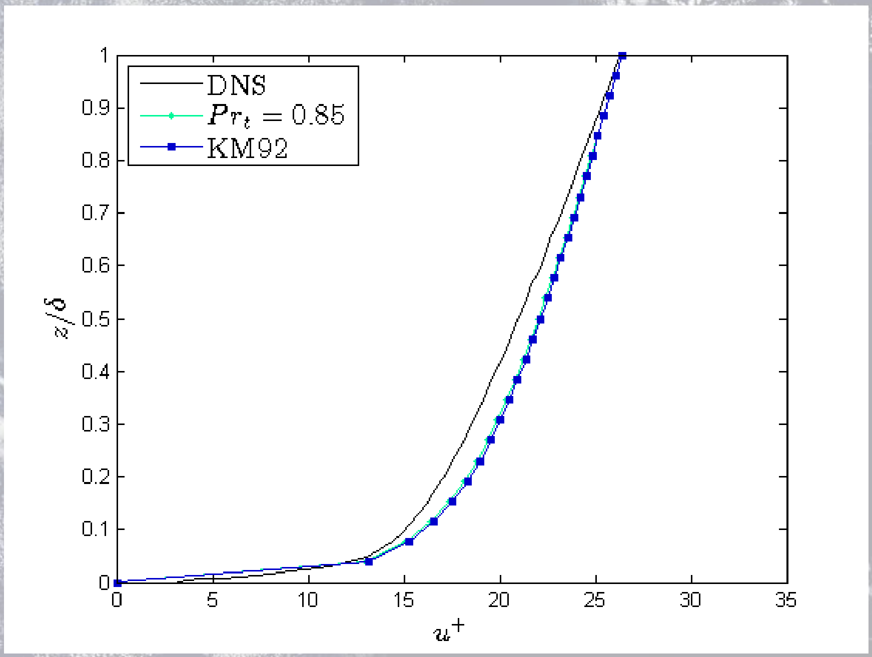

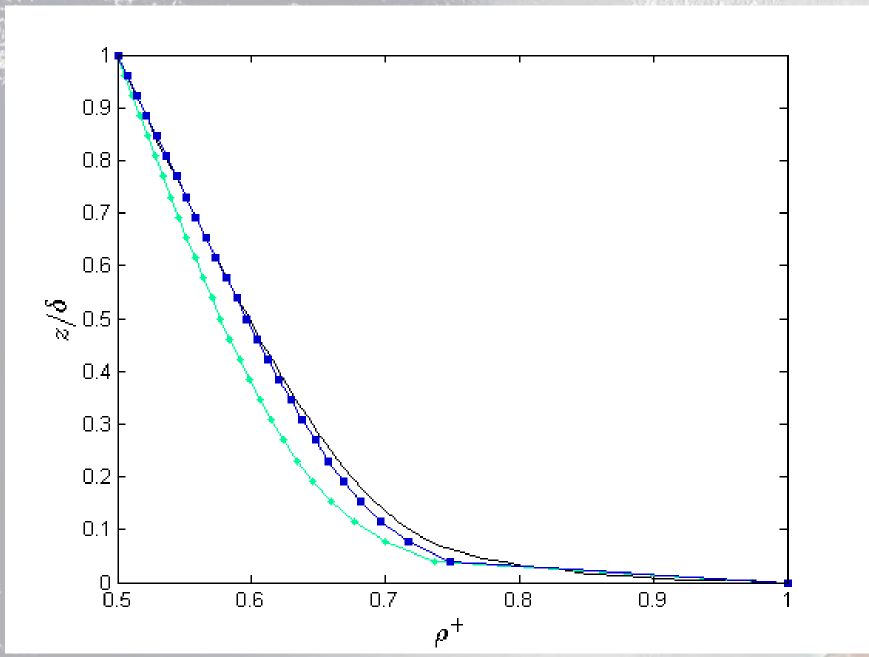

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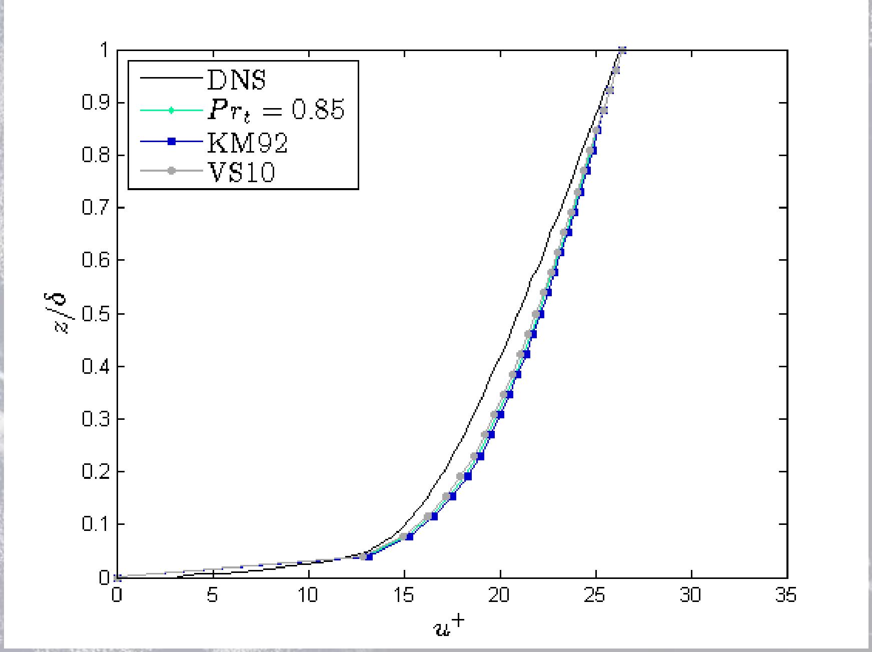
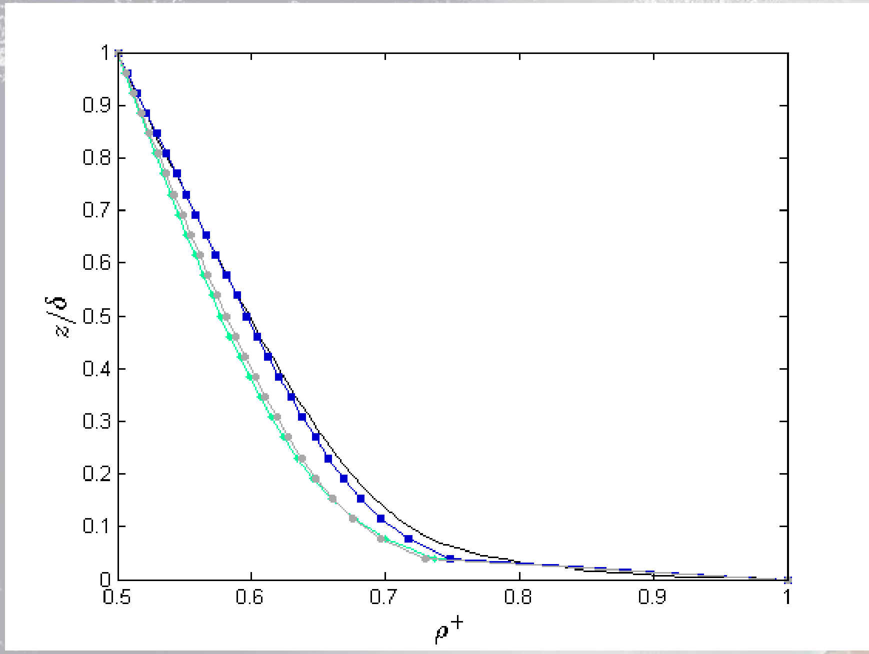
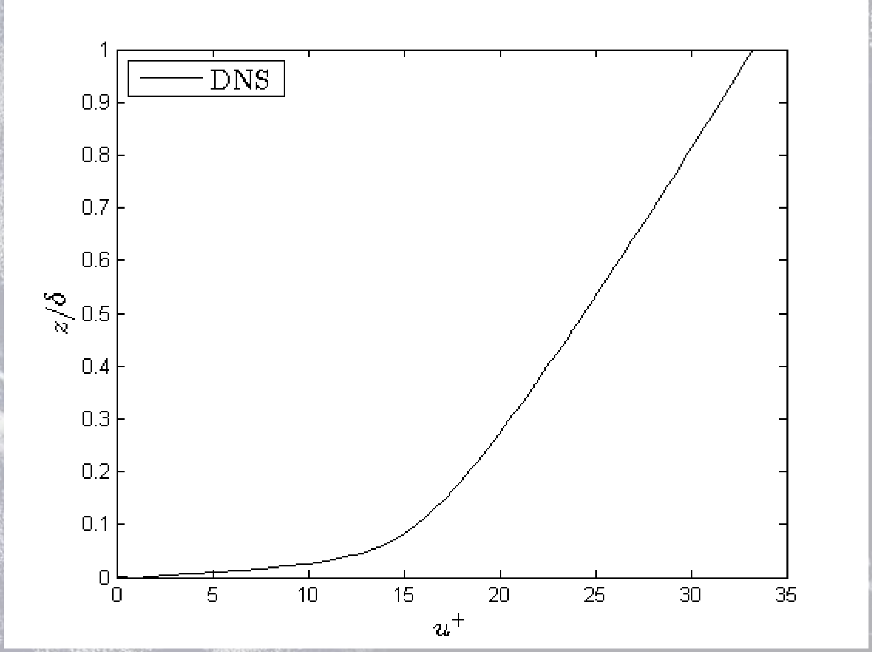
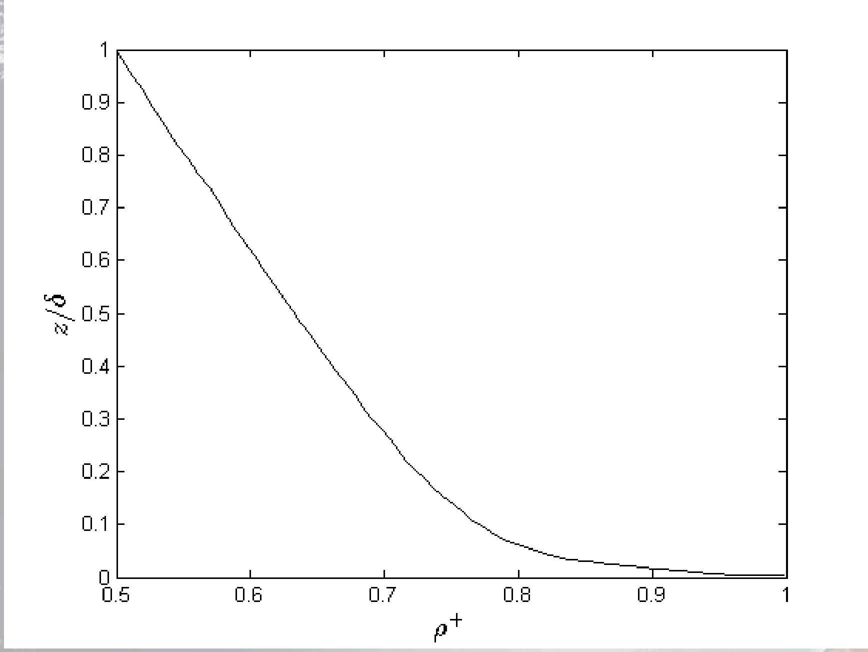




Couette Flow Results

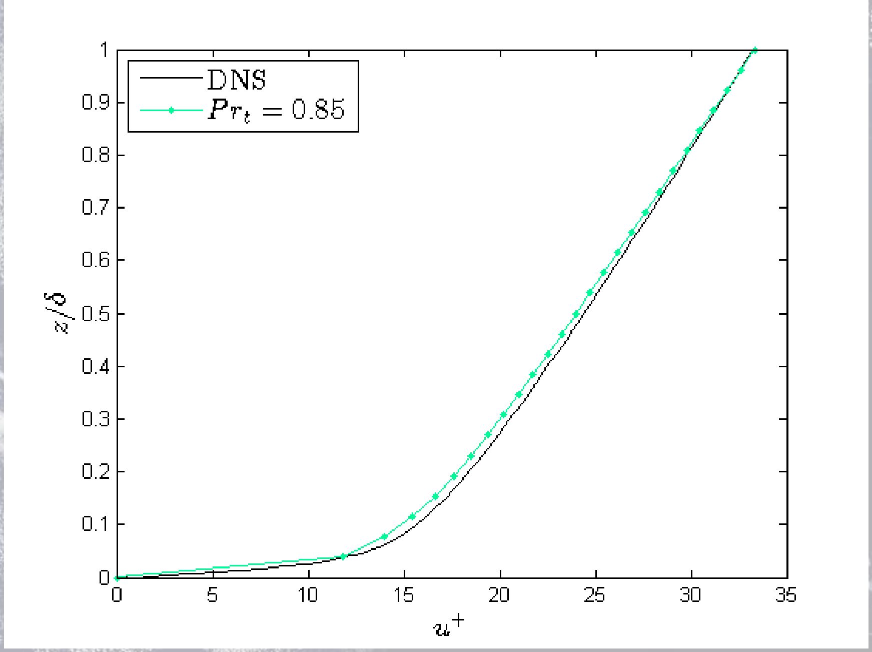
- $Ri_\tau = 167$

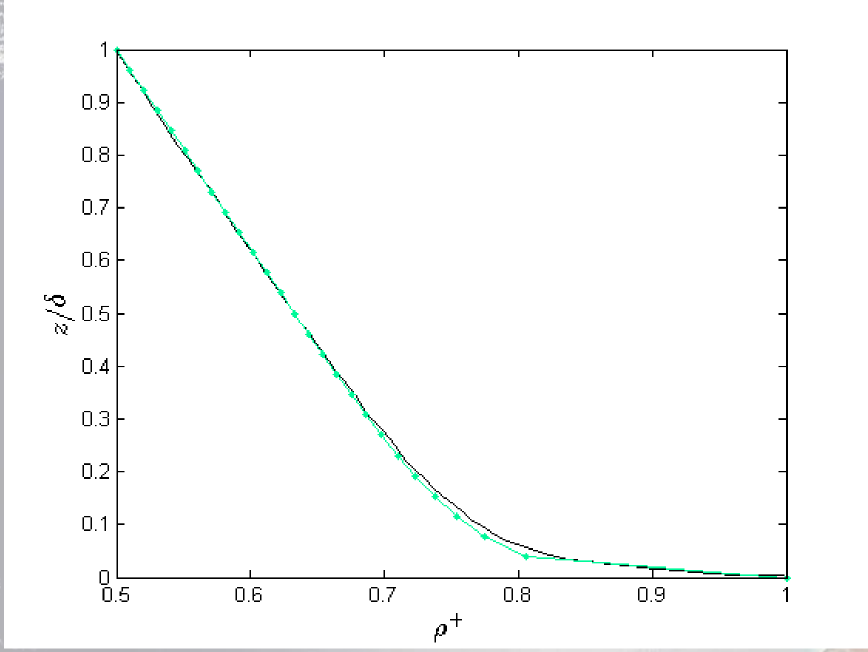




Couette Flow Results

- $Ri_\tau = 167$





Couette Flow Results

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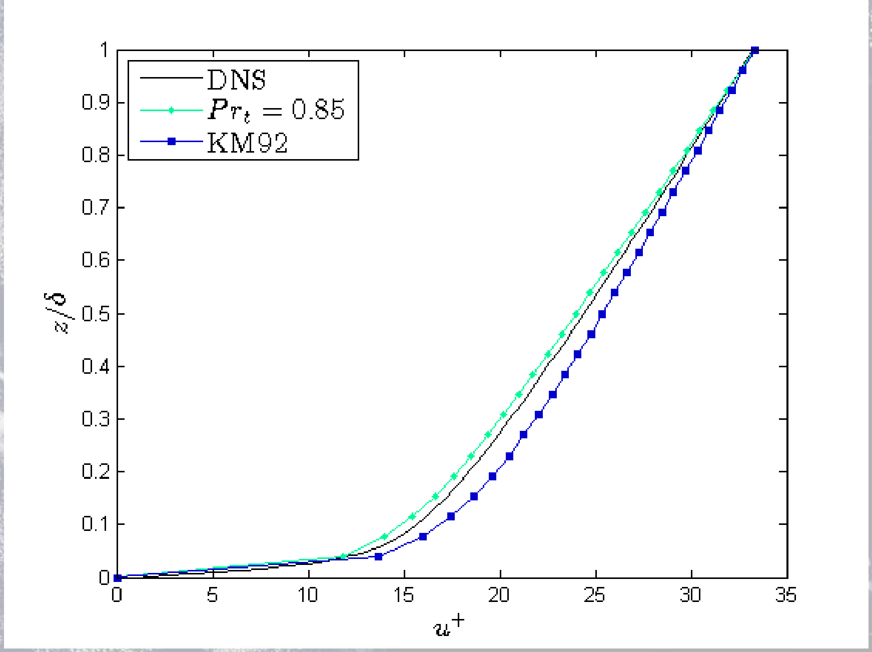

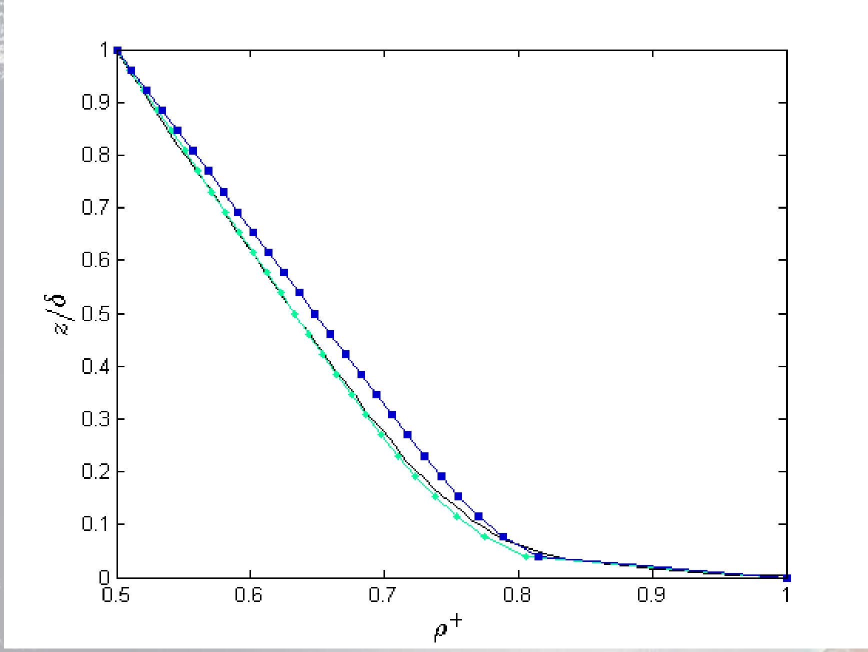

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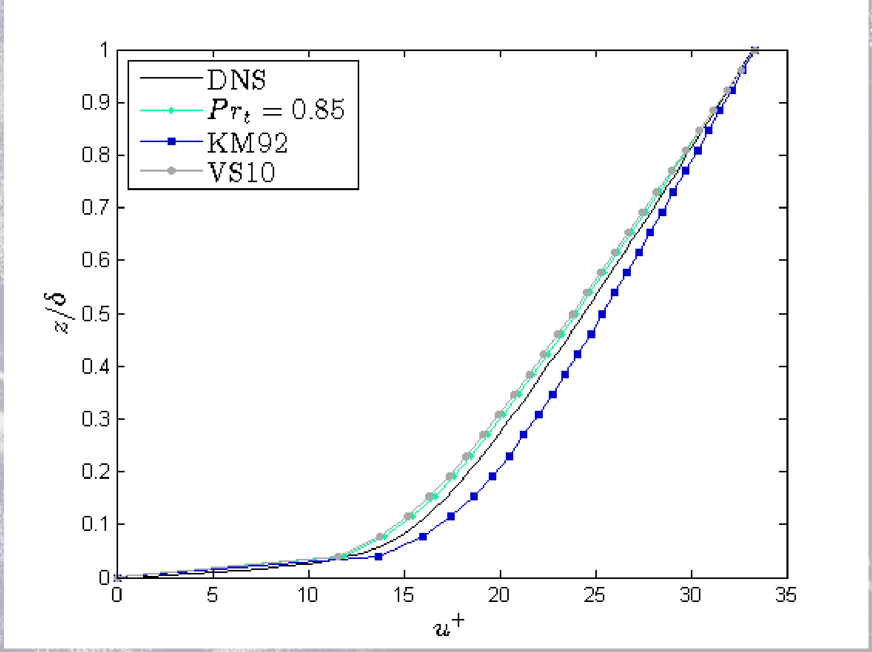
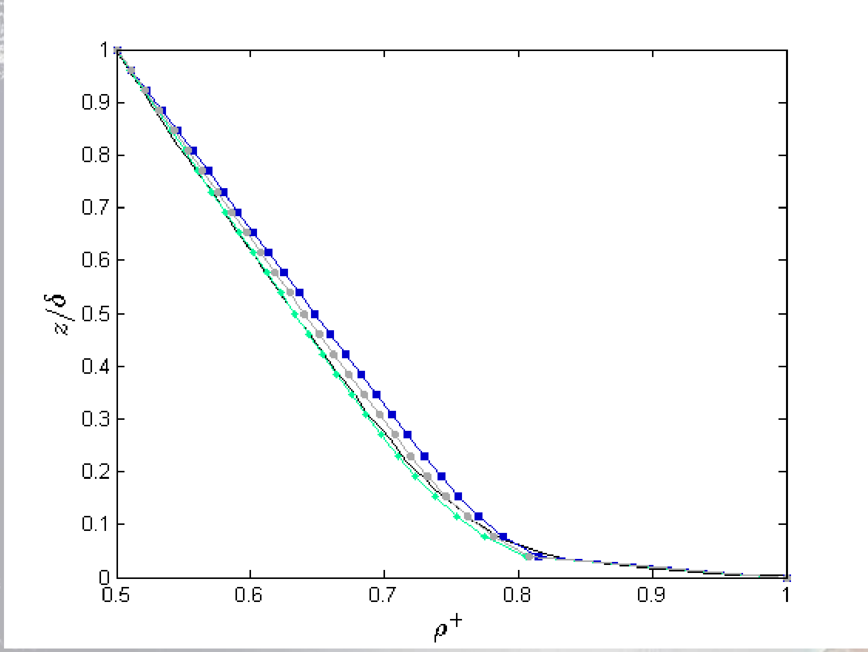
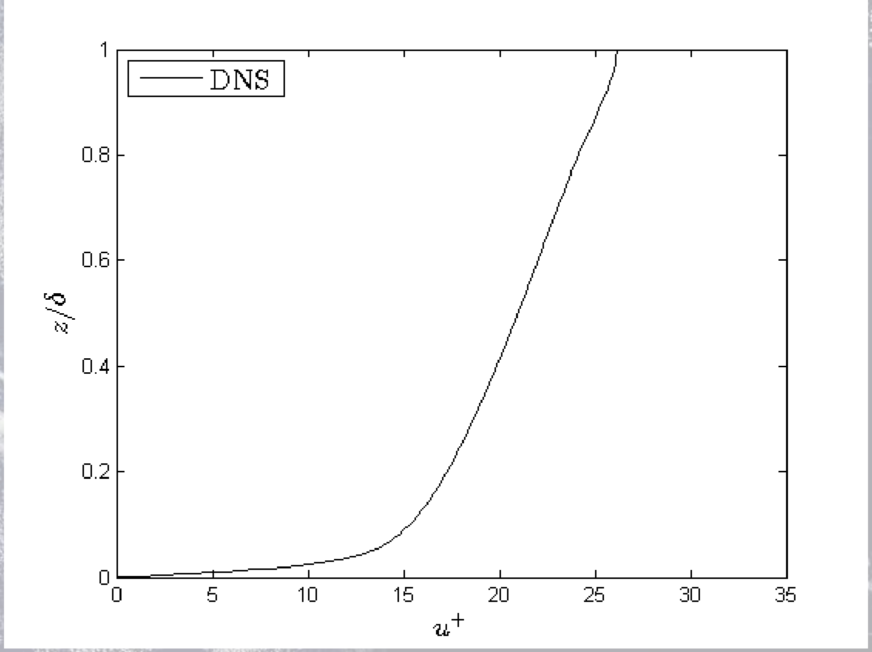
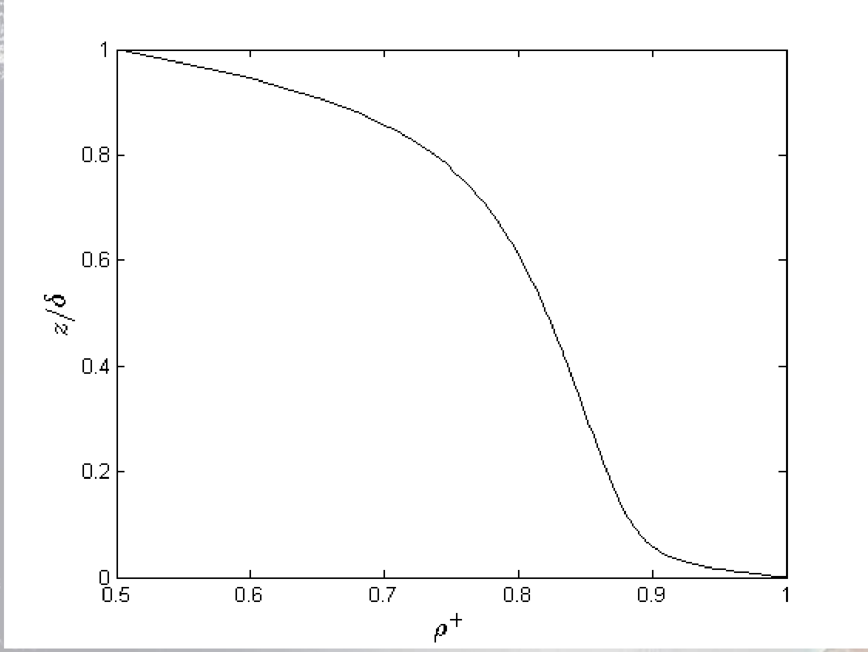




Channel Flow Results

- $Ri_\tau = 60$





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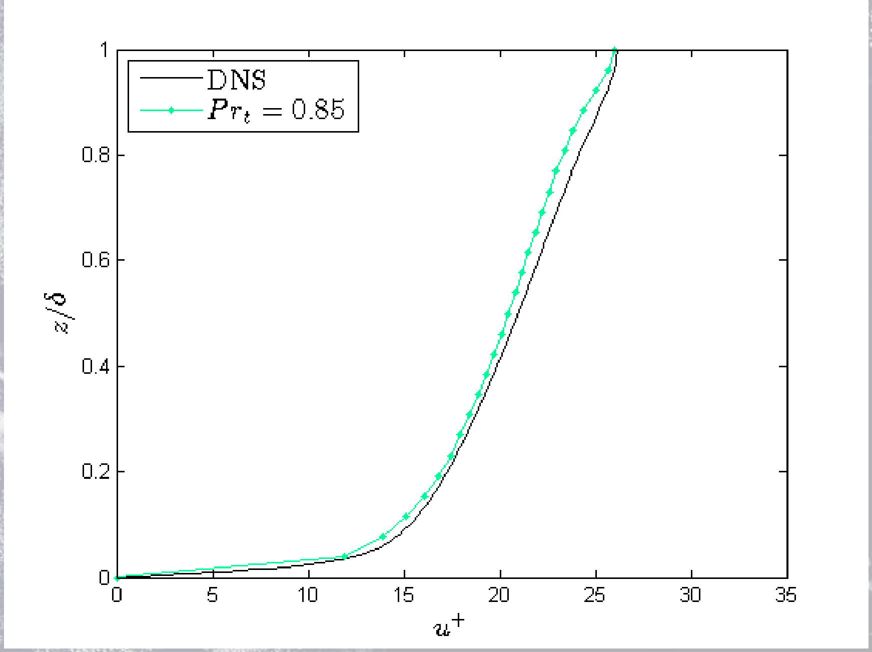

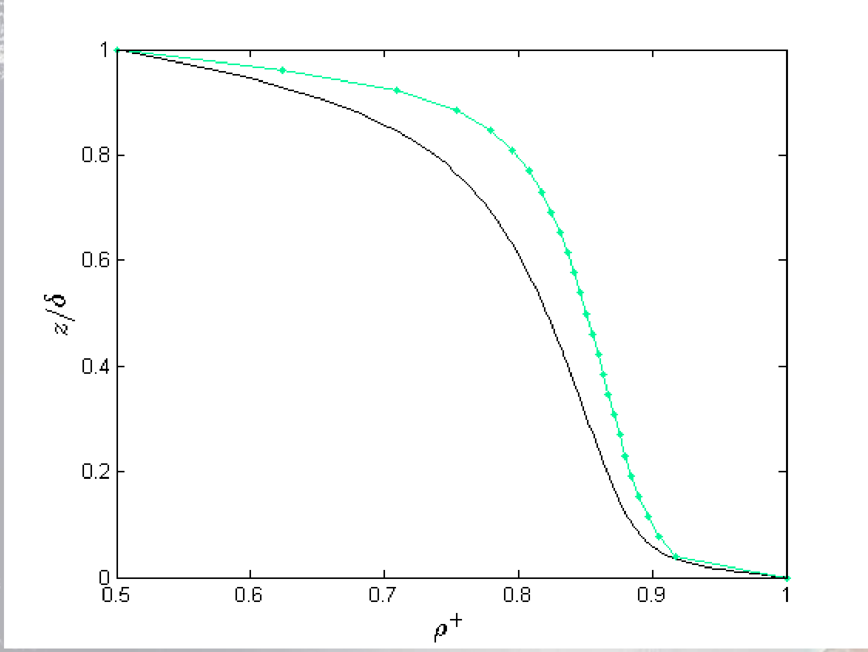

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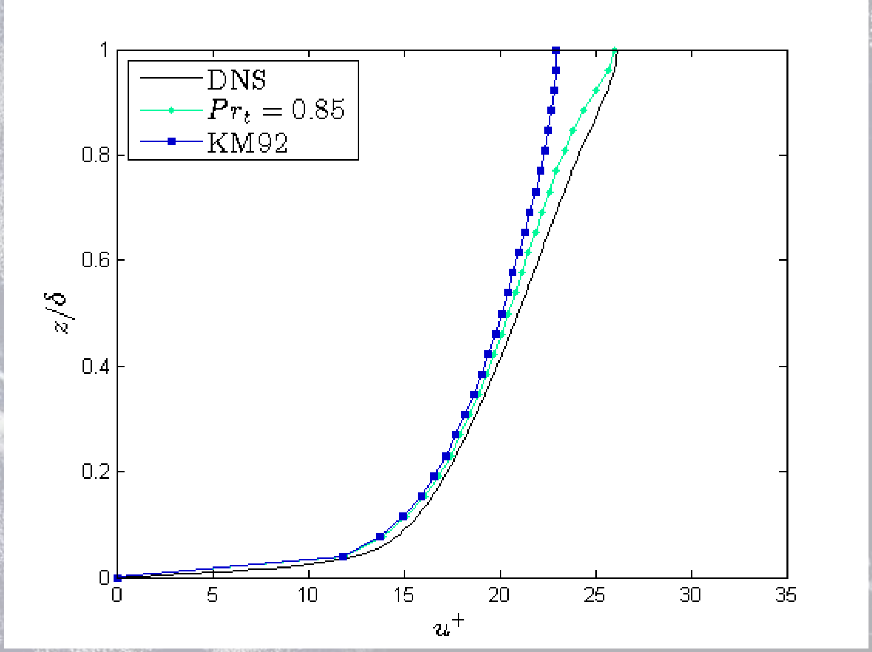

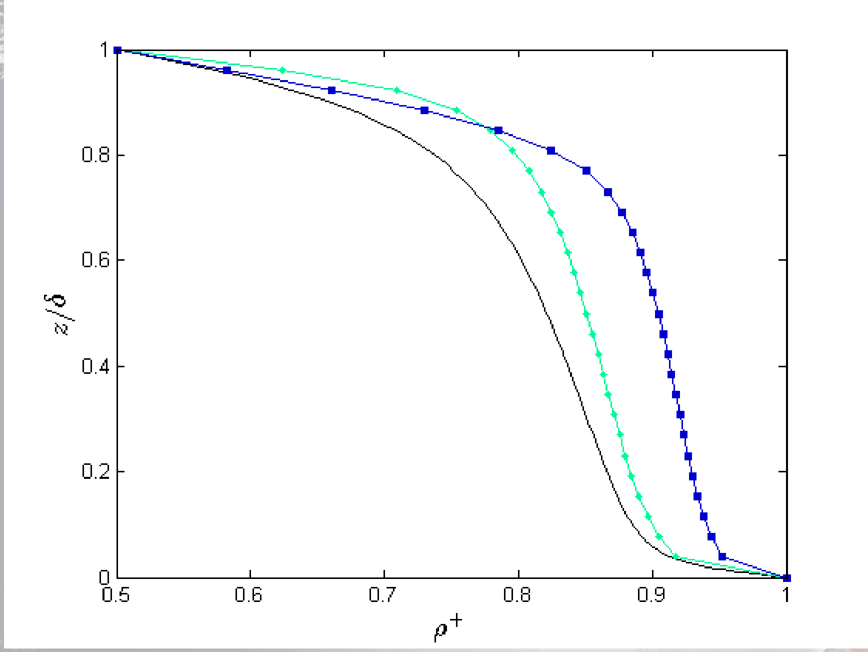

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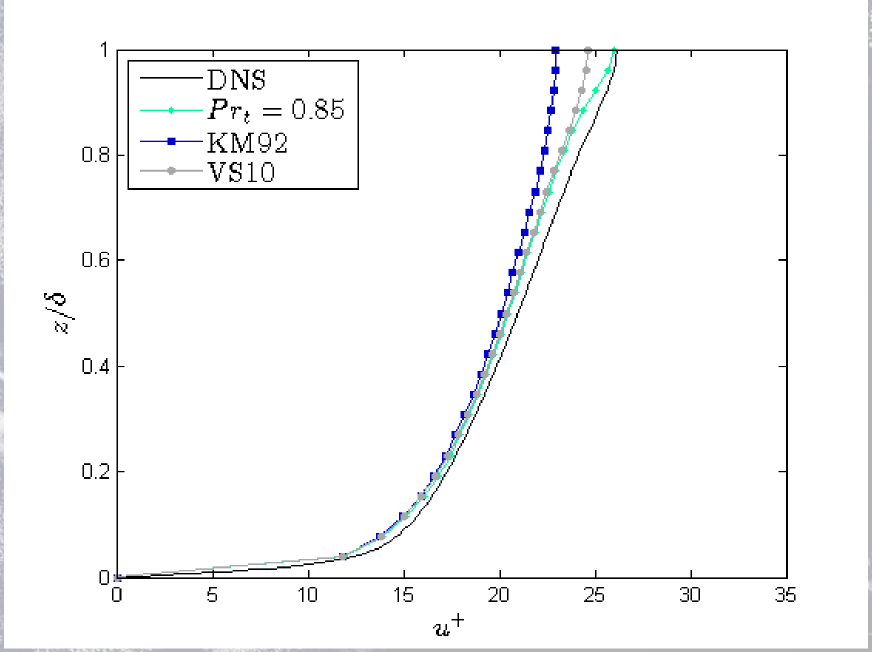
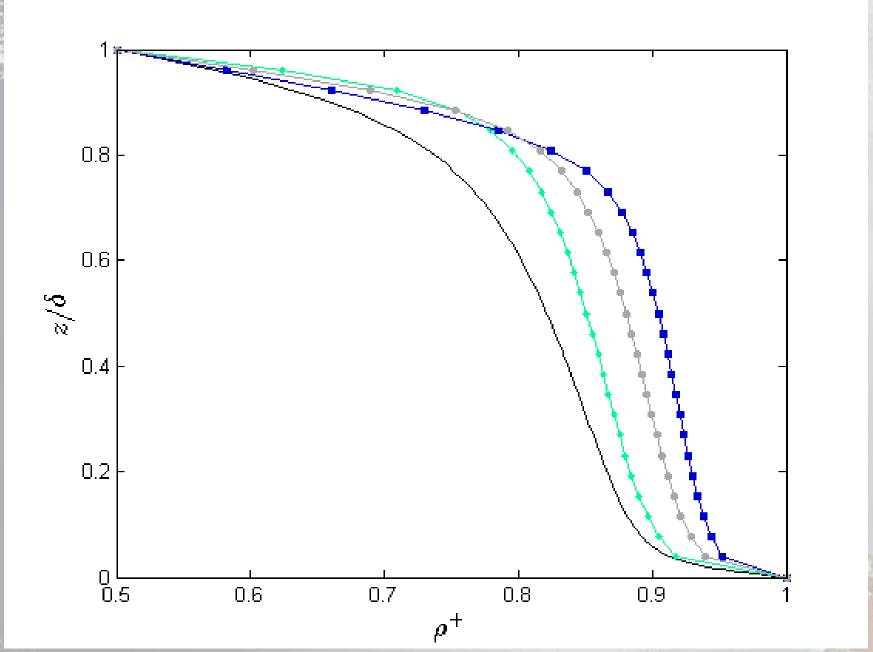


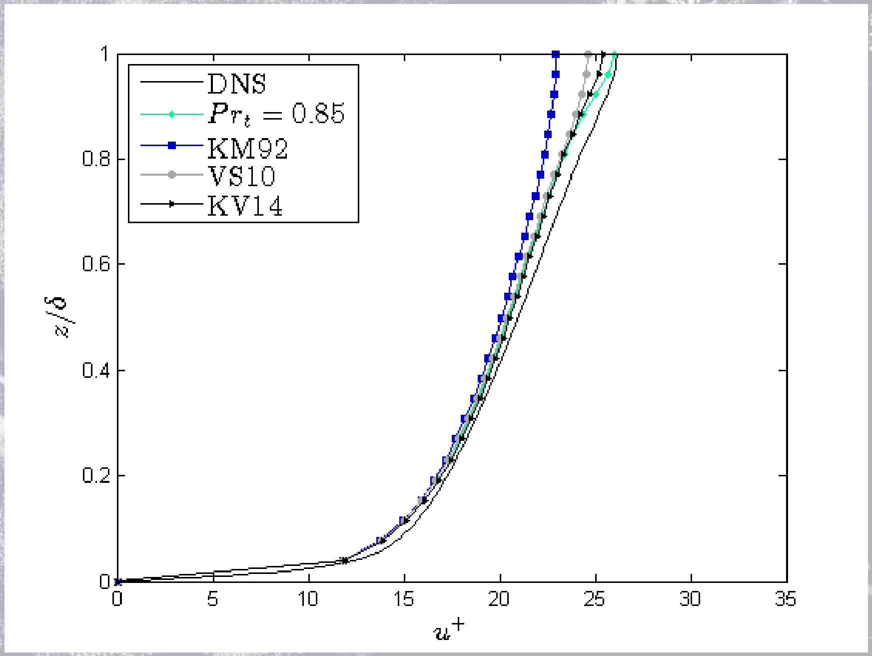


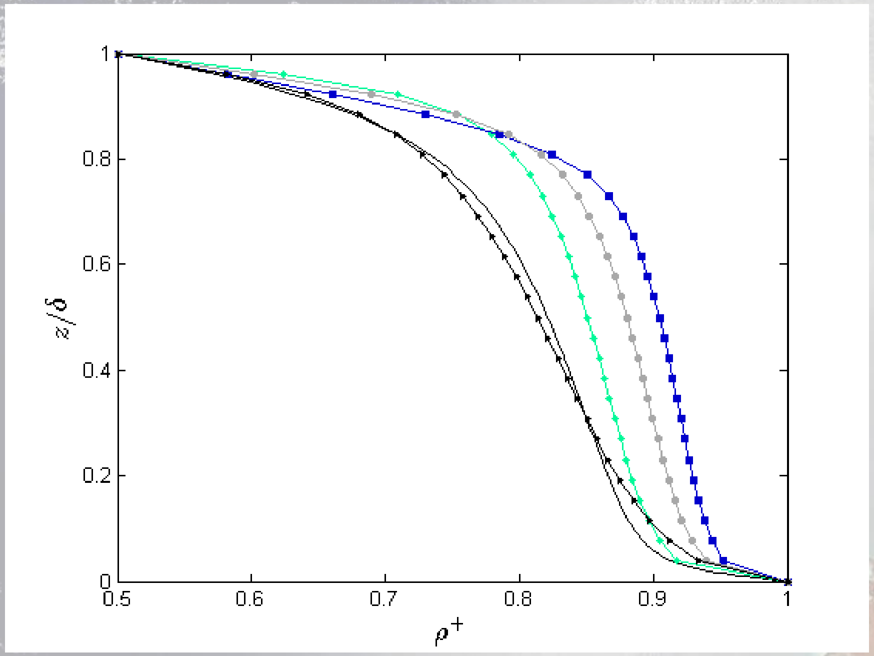
Channel Flow Results

- All Pr_t formulations overestimate mixing of the mean density field
  - $\mathrm{Pr_t}$ formulation of Karimpour & Venayagamoorthy (2014) for wall-bounded flows
  - Linear stress distribution in a channel flow
    - $\tau = \tau_w \left(1 - \frac{z}{D}\right)$
  - Unstratified $\mathrm{Pr_t}$ in a wall bounded flow
    - $\mathrm{Pr_{t0}} = \left(1 - \frac{z}{D}\right) \mathrm{Pr_{twd0}} + \mathrm{Pr_{t0}} \approx 1.1$
  - The turbulent viscosity in the log-law region is given by
    - $\nu_t = \frac{u_\tau^2}{S}\left(1 - \frac{z}{D}\right)$
  - Further assumptions for the log-law region in a wall-bounded flow and subsequent substitutions results in
    - $\mathrm{Pr_t} = \left(1 - \frac{z}{D}\right)\frac{\mathrm{Ri}}{\mathrm{Ri_f}} + \left(1 - \frac{z}{D}\right)\mathrm{Pr_{twd0}} + \mathrm{Pr_{t0}}$ (KV14)
    - Where $\mathrm{Ri_f} = 0.25[1 - \exp(-\gamma \mathrm{Ri})]$

Channel Flow Results

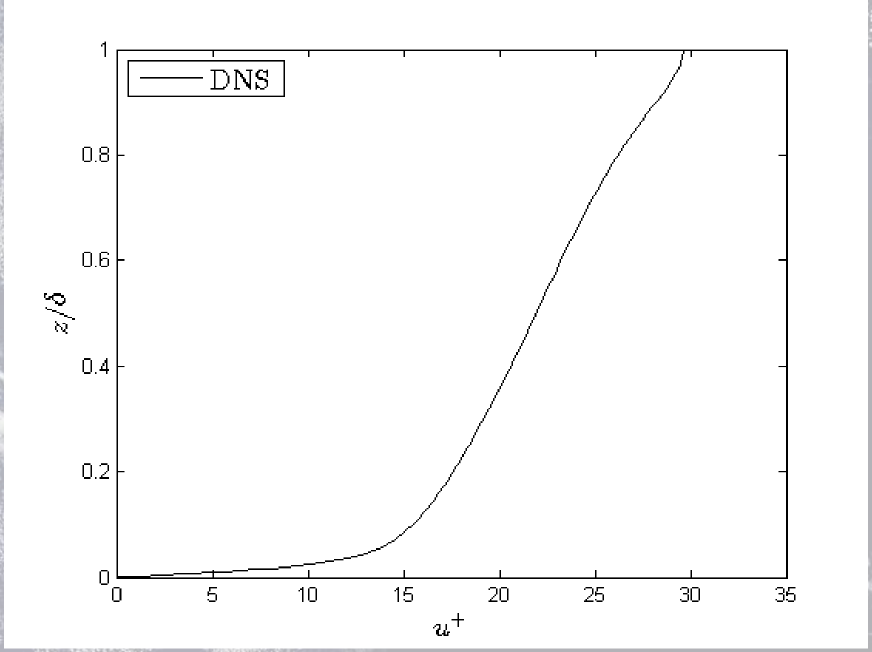
- $Ri_\tau = 60$

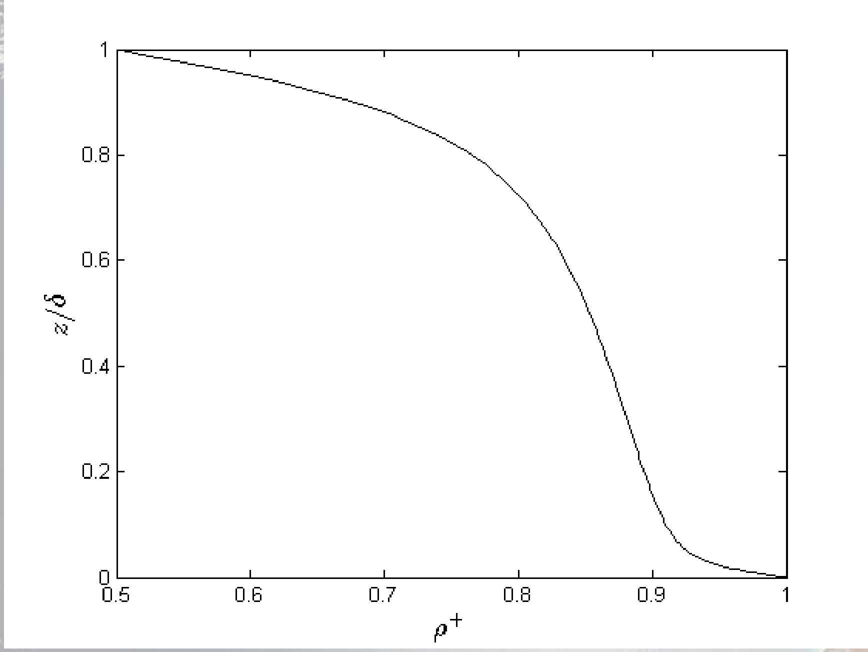




Channel Flow Results

- $Ri_\tau = 120$





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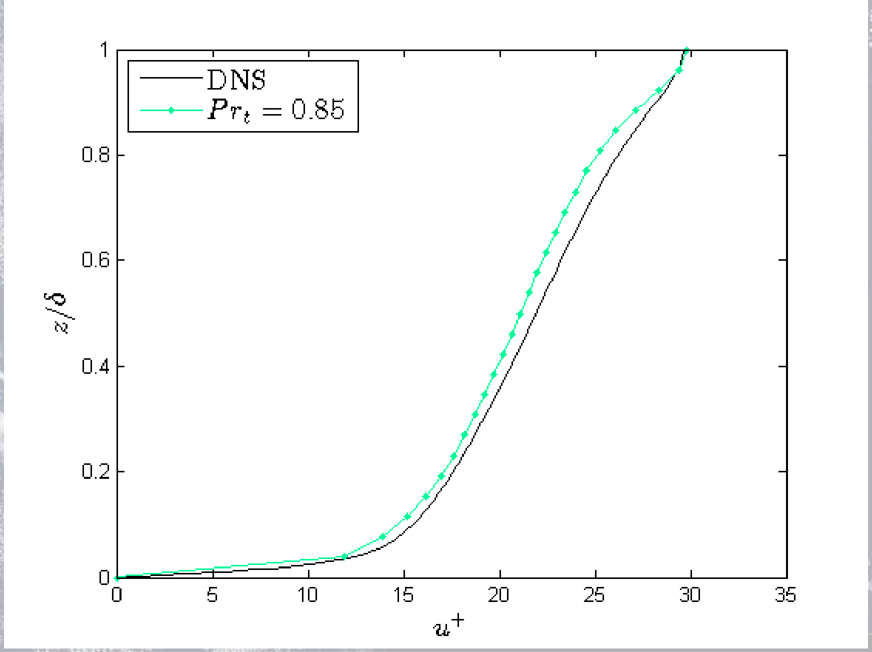

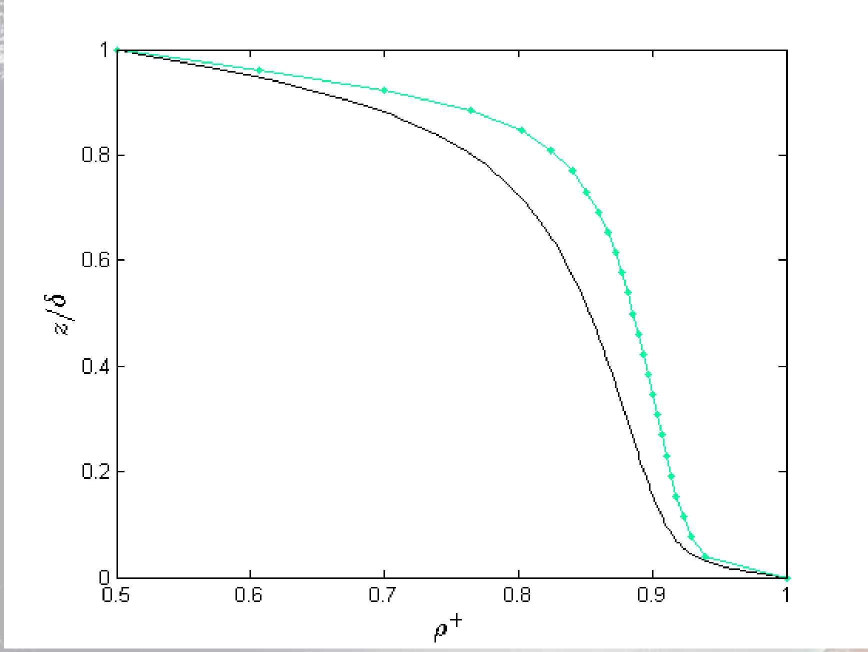

Channel Flow Results

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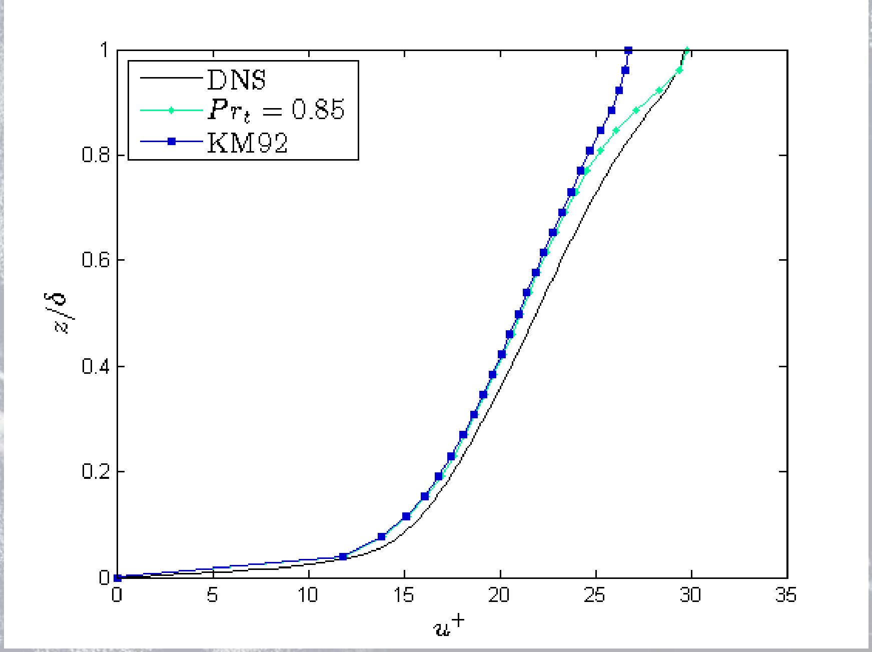

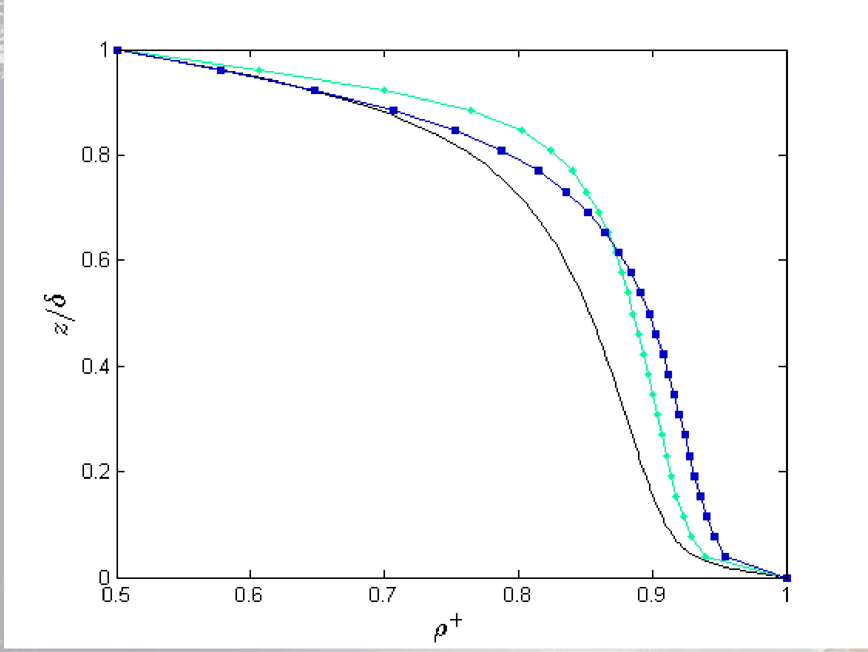

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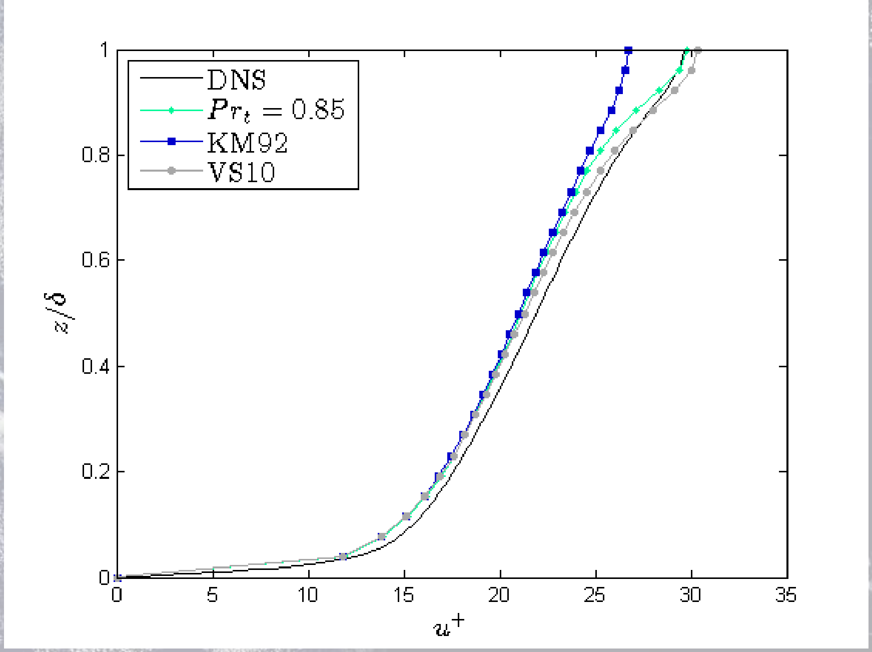

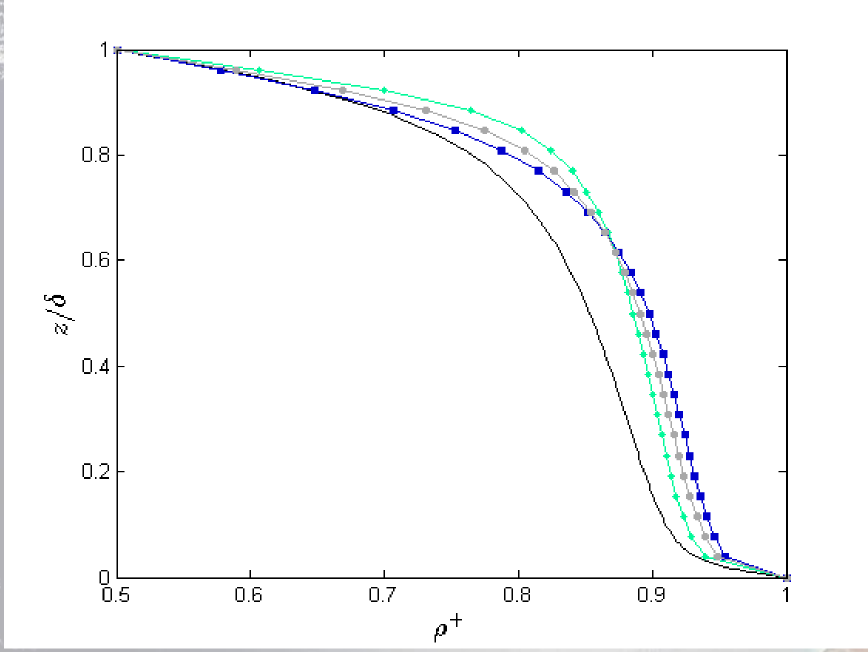

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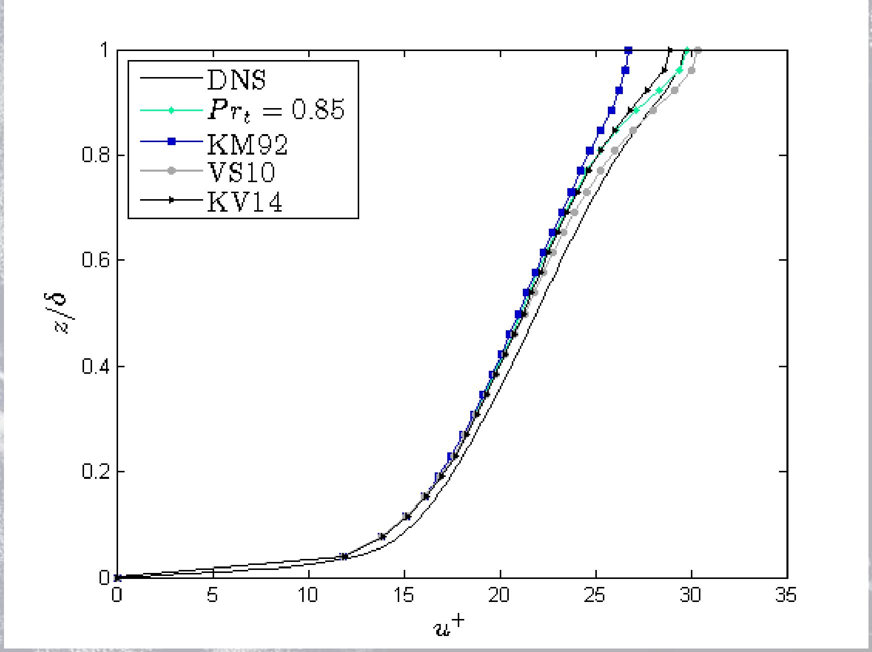
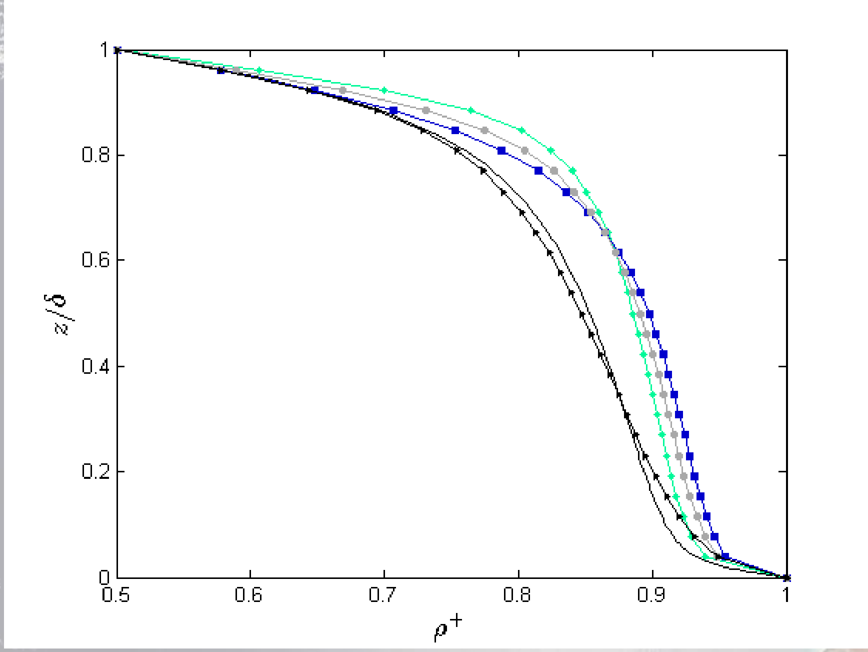
- $Ri_\tau = 120$



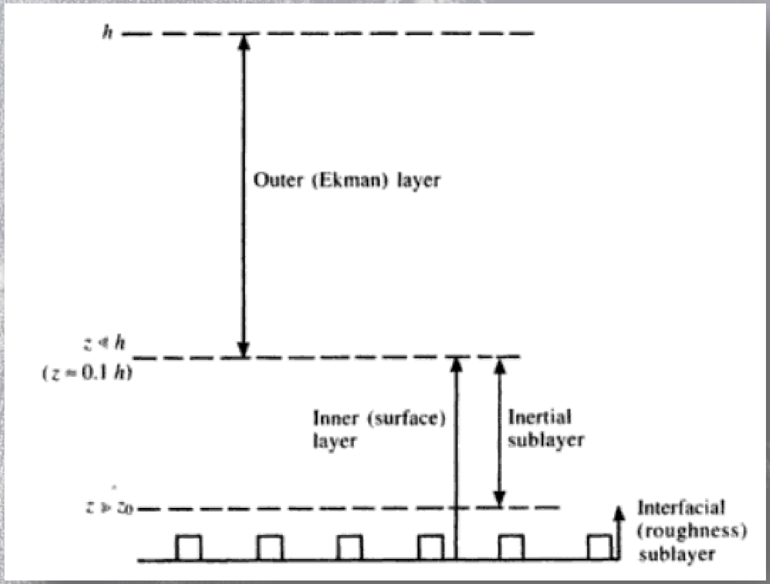


Conclusions

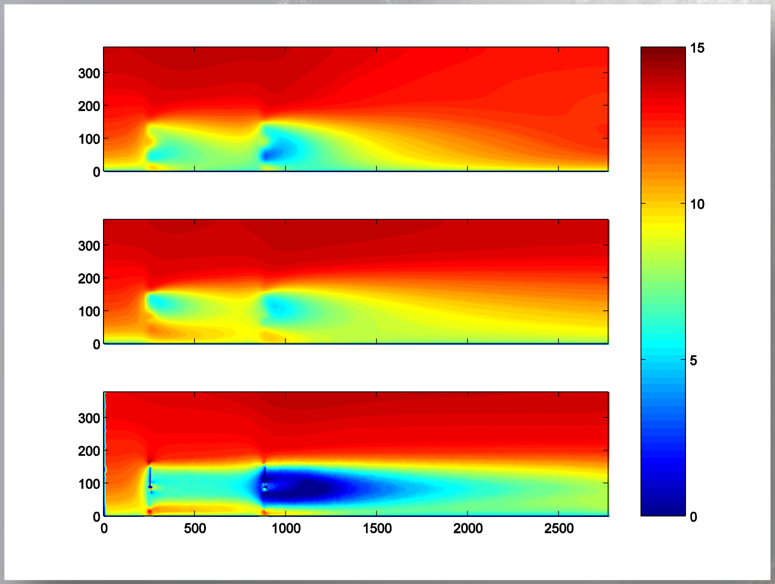
- In stably stratified flows, mean velocity and density fields are evaluated
  - Accurate prediction of mean density proves more difficult than mean velocity
- For Couette flow, constant and homogeneous $\mathrm{Pr_t}$ formulations predict mean velocity and density fields well
- For channel flow, constant and homogeneous $\mathrm{Pr_t}$ formulations predict mean velocity well but over-predict mixing in the mean density field.
  - An inhomogeneous, wall-bounded $\mathrm{Pr_t}$ formulation significantly improves the prediction of mean density

Future Research

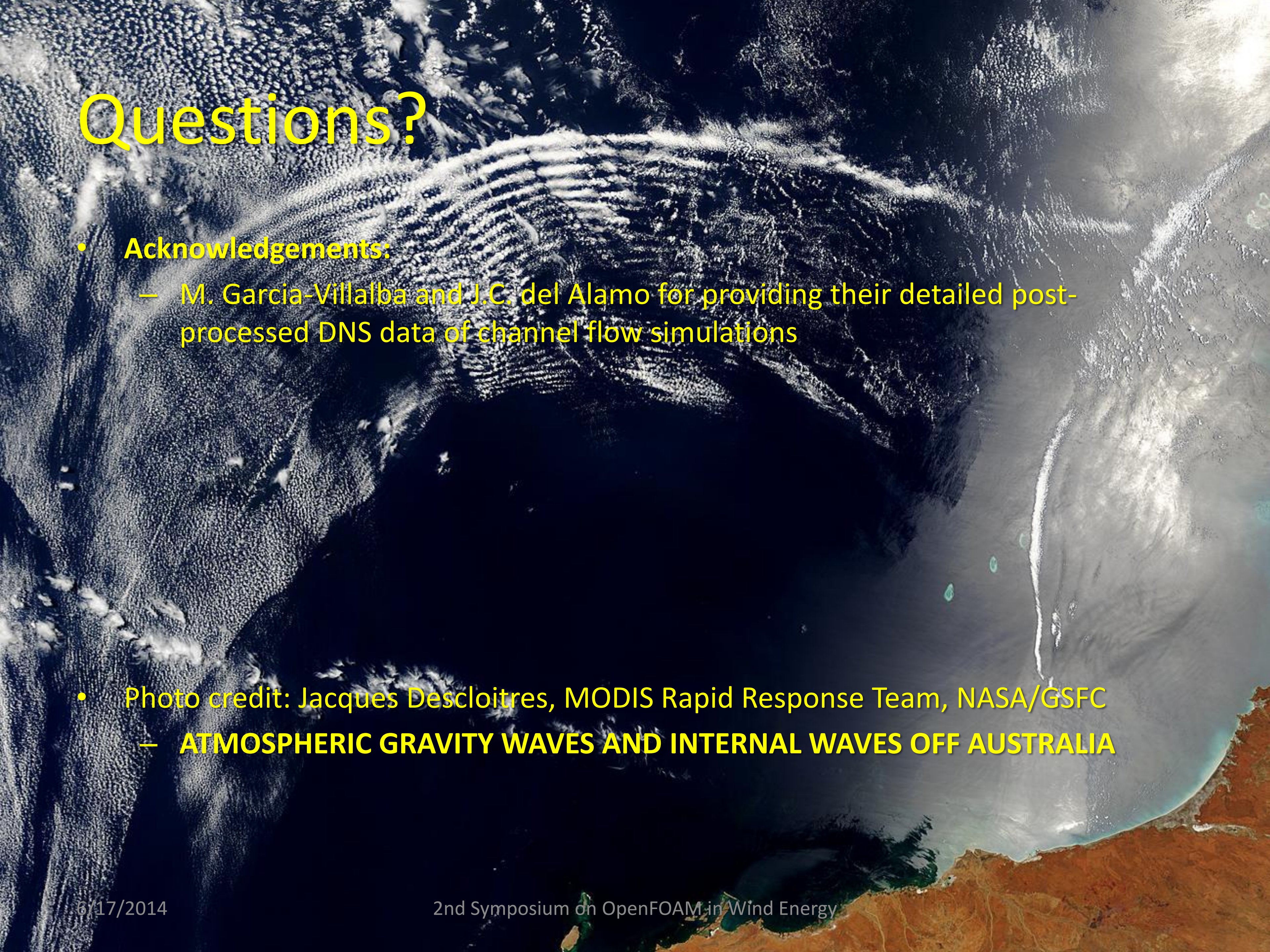
- Moving towards the stable ABL and wind turbine interactions
- Coriolis force (Ekman layer), surface roughness



Garratt (1992)



NREL 5MW Turbine Interactions under neutral conditions



Questions?

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  - M. Garcia-Villalba and J.C. del Alamo for providing their detailed post-processed DNS data of channel flow simulations

- Photo credit: Jacques Descloitres, MODIS Rapid Response Team, NASA/GSFC
  - **ATMOSPHERIC GRAVITY WAVES AND INTERNAL WAVES OFF AUSTRALIA**